

Observational constraints on a cosmological model with variable equation of state parameters for matter and dark energy

Suresh Kumar ^{*} Lixin Xu [†]

July 31, 2012

Abstract

In this work we consider a spatially homogeneous and flat Friedmann-Robertson-Walker (FRW) space-time filled with non-interacting matter and dark energy components. The equation of state (EoS) parameters of the two sources are varied phenomenologically in terms of scale factor of the FRW space-time in such a way that the evolution of the Universe takes place from the early radiation-dominated phase to the present dark energy-dominated phase. We constrain the derived model in two cases with the latest astronomical observations, and discuss the best fit model parameters in detail. First, we explore a special case of the model with WMAP+BAO+H0 observations by synchronizing the model with the Λ CDM model at the present epoch. An interesting point that emerges from this observational analysis is that the model is not only consistent with the Λ CDM predictions at the present epoch but also is indistinguishable from the Λ CDM model in revealing the future dynamics of the Universe. In the second case, we find observational constraints on general class of the model from Supernova+BAO observations. The derived model, in the general case, predicts age of the Universe, Hubble constant, density parameters and equation of state parameter of dark energy consistent with the ones obtained from seven year WMAP observations. The model advocates cosmological constant as a candidate of dark energy, which is consistent with the WMAP observations. Finally, we conclude that the derived model offers a unified description of the evolution of Universe from the early radiation-dominated phase to the present dark energy-dominated phase in accord with the current astronomical observations. The model is physically viable and is applicable to the real Universe.

Keywords: FRW space-time · Accelerating Universe · Varying equation of state parameters · Dark energy · Cosmological Constant.

1 Introduction

It is not a matter of debate now whether the Universe is accelerating at the present epoch since it is strongly supported by various astronomical probes of complementary nature such as type Ia supernovae data (SN Ia) [1, 2], galaxy redshift surveys [3], cosmic microwave background radiation (CMBR) data [4, 5] and large scale structure [6]. Observations also suggest that there had been a transition of the Universe from the earlier deceleration phase to the recent acceleration phase [7]. We do not have a fundamental understanding of the root cause of the accelerating expansion of the Universe. We label our ignorance with the term “Dark Energy” (DE), which is assumed to permeate all of space and increase the rate of expansion of the Universe [8]. On the other hand, the inclusion of DE into the prevailing theory of cosmology has been enormously successful in resolving numerous puzzles that plagued this field for many years. For example, with prior cosmological models, the Universe appeared to be younger than its oldest stars. When DE is included in the model, the problem goes away.

^{*}Department of Applied Mathematics, Delhi Technological University, Delhi-110 042, India. E-Mail: sukuyd@gmail.com

[†]Institute of Theoretical Physics, School of Physics & Optoelectronic Technology, Dalian University of Technology, Dalian, 116024, P. R. China . E-Mail: lxxu@dlut.edu.cn

The most recent WMAP observations indicate that DE accounts for around three fourth of the total mass energy of the universe [9]. However, the nature of DE is still unknown and various cosmological probes on theoretical and experimental fronts are in progress to resolve this problem. The simplest candidate for the DE is the cosmological constant (Λ) or vacuum energy since it fits the observational data well. During the cosmological evolution, the Λ -term has the constant energy density and pressure $p_{de} = -\rho_{de}$. However, one has the reason to dislike the cosmological constant since it always suffers from the theoretical problems such as the “fine-tuning” and “cosmic coincidence” puzzles [10]. That is why, the different forms of dynamically changing DE with an effective equation of state (EoS), $w_{de} = p_{de}/\rho_{de} < -1/3$, have been proposed in the literature. Other possible forms of DE include quintessence ($w_{de} > -1$) [11], phantom ($w_{de} < -1$) [12] etc. Using the first year Sloan Digital Sky Survey-II (SDSS-II) supernova results, Lampeitl et al. [13] found that $-0.99 < w_{de} < -0.65$ while the seven year WMAP results put w_{de} in the range $-1.55 < w_{de} < -0.7$ (see, Jarosik et al. [14]).

In cosmology, the evolution of the Universe is described by the Einstein’s field equations together with an EoS ($p = w\rho$) for the matter content. Usually the field equations are solved and analyzed separately for different epochs, i.e., for inflationary phase, radiation-dominated phase and matter-dominated phase. Some authors have presented unified solutions for these epochs. For instance, Israelit and Rosen [15, 16] presented a unified EoS, where the pressure varies continuously from pre-matter period ($p = -\rho$) to the radiation-dominated phase ($p = \rho/3$) and then radiation to matter-dominated period ($p = 0$) for spatially closed, flat and open FRW models. They also described a transition between pre-matter to radiation and radiation to matter-dominated epoch. Similarly, Carvalho [17] studied a flat FRW model, where the Universe undergoes a transition from an inflationary phase to a radiation-dominated phase. However, in the models presented by Israelit and Rosen [15, 16], only ordinary matter was taken into account while Carvalho [17] studied the model with cosmological constant. In a recent paper [18], a physically reasonable and mathematically tractable cosmological model is studied by assuming time-varying EoS parameters for matter and DE. Here, we intend to study a cosmological model based on a special case of EoS parameters proposed in [18].

In this paper, we consider non-interacting matter (dark matter plus baryonic) and DE energy components within the framework of a spatially homogeneous and flat FRW space-time in general relativity. Following Ref. [18], the EoS parameters of the two sources have been varied phenomenologically in terms of scale factor of the FRW space-time in such a way that the evolution of the Universe takes place from the early radiation-dominated phase to the present DE-dominated phase. The paper is structured as follows. The model and field equations are presented in Section 2. In Section 3, we present the cosmological model with time-varying EoS parameters for the matter and DE components. In Section 4, we explore a special case of the model with WMAP+BAO+H0 observations by synchronizing the model with the Λ CDM model at the present epoch. Section 5 is devoted to find the best fit model constrained with latest SN Ia+BAO observations. The findings of the paper are summarized in Section 6.

2 Model and field equations

We consider a spatially homogeneous and flat FRW line element that applies to the real Universe. It is written as

$$ds^2 = -dt^2 + a^2(t) [dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)], \quad (1)$$

where $a(t)$ is cosmic scale factor and other symbols have their usual meanings.

The Einstein’s field equations in case of a mixture of matter and DE components, in the units $8\pi G = c = 1$, read as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -T_{\mu\nu}, \quad (2)$$

where $T_{\mu\nu} = T_{\mu\nu}^{(m)} + T_{\mu\nu}^{(de)}$ is the overall energy momentum tensor with $T_{\mu\nu}^{(m)}$ and $T_{\mu\nu}^{(de)}$ as the energy

momentum tensors of matter and DE, respectively. These are given by

$$\begin{aligned} T_{\nu}^{(m)\mu} &= \text{diag} [-\rho_m, p_m, p_m, p_m] \\ &= \text{diag} [-1, w_m, w_m, w_m] \rho_m \end{aligned} \quad (3)$$

and

$$\begin{aligned} T_{\nu}^{(de)\mu} &= \text{diag} [-\rho_{de}, p_{de}, p_{de}, p_{de}] \\ &= \text{diag} [-1, w_{de}, w_{de}, w_{de}] \rho_{de} \end{aligned} \quad (4)$$

where ρ_m and p_m are, respectively the energy density and pressure of the matter fluid while $w_m = p_m/\rho_m$ is its EoS parameter. Similarly, ρ_{de} and p_{de} are, respectively the energy density and pressure of the DE fluid while $w_{de} = p_{de}/\rho_{de}$ is the corresponding EoS parameter.

In a comoving coordinate system, the field equations (2) for the space-time (1), in case of (3) and (4), read as

$$2\dot{H} + 3H^2 = -w_m\rho_m - w_{de}\rho_{de}, \quad (5)$$

$$3H^2 = \rho_m + \rho_{de}. \quad (6)$$

Here an over dot indicates ordinary derivative with respect to t , and $H = \dot{a}/a$ is the Hubble parameter. The energy conservation equation $T^{\mu\nu}_{;\nu} = 0$ yields

$$\dot{\rho}_m + 3(1 + w_m)\rho_m H + \dot{\rho}_{de} + 3(1 + w_{de})\rho_{de} H = 0. \quad (7)$$

We assume that the matter and DE components are non-interacting. Therefore, the energy momentum tensors of the two sources may be conserved separately.

The energy conservation equation $T^{(m)\mu\nu}_{;\nu} = 0$, of the matter fluid leads to

$$\dot{\rho}_m + 3(1 + w_m)\rho_m H = 0, \quad (8)$$

whereas the energy conservation equation $T^{(de)\mu\nu}_{;\nu} = 0$, of the DE component yields

$$\dot{\rho}_{de} + 3(1 + w_{de})\rho_{de} H = 0. \quad (9)$$

3 Cosmology with varying EoS parameters

In order to construct a model for unified description of the evolution of the Universe from the early radiation-dominated phase to the recent DE-dominated phase, we assume the following forms for the EoS parameters of matter and DE (see [18] for more general EoS parameters):

$$w_m = \frac{1}{3(x^\alpha + 1)}, \quad (10)$$

$$w_{de} = \frac{\bar{w}x^\alpha}{x^\alpha + 1}, \quad (11)$$

where $x = a/a_*$ with a_* being some reference value of a . Further, α is some positive constant parameter while \bar{w} is a negative constant.

Substituting (10) into (8), we find

$$\rho_m = \frac{C_1 (x^\alpha + 1)^{1/\alpha}}{x^4}, \quad (12)$$

$$p_m = \frac{C_1 (x^\alpha + 1)^{(1-\alpha)/\alpha}}{3x^4}, \quad (13)$$

where C_1 is a positive constant of integration.

Similarly, substituting (11) into (9), we obtain

$$\rho_{de} = \frac{C_2 (x^\alpha + 1)^{-3\bar{w}/\alpha}}{x^3}, \quad (14)$$

$$p_{de} = \frac{\bar{w}C_2 (x^\alpha + 1)^{-(3\bar{w}+\alpha)/\alpha}}{x^{(3-\alpha)}}, \quad (15)$$

where C_2 is a positive constant of integration.

- For $x \ll 1$, we have

$$\rho_m \approx \frac{C_1}{x^4}, \quad p_m = \frac{1}{3}\rho_m, \quad \rho_{de} \approx \frac{C_2}{x^3},$$

which means that for $x \ll 1$ matter dominates over DE, as expected. Besides, in this limit we find

$$p_{de} \approx \frac{\bar{w}C_2}{3x^{(3-\alpha)}}.$$

- For $x \gg 1$, we get

$$\rho_m \approx \frac{C_1}{x^3}, \quad p_m = \frac{C_1}{3x^{(3+\alpha)}}.$$

Since $\alpha > 0$, the matter pressure decreases with the expansion of the Universe much faster than the density. Thus, $\rho_m \gg p_m$ as required by a matter dominated Universe. Note that ρ_m decreases exactly as in cosmological dust models.

In addition, in this limit we obtain

$$p_{de} = \bar{w}\rho_{de}, \quad \rho_{de} \approx \frac{C_2}{x^{3(1+\bar{w})}}.$$

Since $\bar{w} < 0$, it follows that $\rho_{de} \gg \rho_m$ and $|p_{de}| \gg p_m$ for $x \gg 1$.

Now from (6), it follows that

$$H = \sqrt{\frac{C_1 (x^\alpha + 1)^{1/\alpha}}{3x^4} + \frac{C_2 (x^\alpha + 1)^{-3\bar{w}/\alpha}}{3x^3}}. \quad (16)$$

Using (10)-(12), (14) and (16) we find that (5) is satisfied identically, as expected. For a detailed classical treatment of the above equations in case of more general EoS parameters, the reader is referred to Ref. [18]. Here, we are interested in finding the observational constraints on the cosmological model in hand.

The effective EoS parameter is obtained as

$$w_{eff} = \frac{p_{eff}}{\rho_{eff}} = \frac{p_m + p_{de}}{\rho_m + \rho_{de}} = \frac{3C_2\bar{w}x^{1+\alpha} + C_1(1+x^\alpha)^{\frac{1+3\bar{w}}{\alpha}}}{3(1+x^\alpha) \left[C_2x + C_1(1+x^\alpha)^{\frac{1+3\bar{w}}{\alpha}} \right]}. \quad (17)$$

In terms of the cosmological redshift z we can write $a = a_0/(1+z)$, where a_0 is the present day value of a , which corresponds to $z = 0$. Thus,

$$x = \frac{1+z_*}{1+z}. \quad (18)$$

It follows that x varies from 0 to ∞ as z varies from ∞ to -1 .

In the following section, we consider a special class of the proposed model and compare it with the Λ CDM model by subjecting the model to WMAP+BAO+H0 observations.

4 Observational constraints on a special class of the model

Table 1 shows the variation of the EoS parameters as z varies from ∞ to -1 .

Table 1: Extreme values of EoS parameters

EoS Parameter	$z \rightarrow \infty$	$z \rightarrow -1$
w_m	$1/3$	0
w_{de}	0	\bar{w}
w_{eff}	$1/3$	\bar{w}

It is interesting to observe that the constant \bar{w} is decisive in future dynamics of the Universe. Also this constant is at our discretion. Thus, if we choose $-1 < \bar{w} < -1/3$, the dynamically evolving DE will never cross the phantom divide line (PDL) ($w_{de} = -1$) and the accelerated expansion of the Universe will continue in future with quintessence form of DE. If we set $\bar{w} < -1$, the PDL will be crossed and the Universe will enter the phantom regime. Choosing $\bar{w} = -1$, we ensure that dynamics of the future Universe will be purely governed by cosmological constant. In what follows we shall carry on with the choice $\bar{w} = -1$. Also, in the proposed model it seems reasonable to interpret a_* as the value of a for which $\rho_m = \rho_{de}$. Then, from (12) and (14) it follows that $C_1 = C_2 2^{2/\alpha}$.

Taking into account the above considerations and restoring the SI units with $M_P^2 = \hbar c / 8\pi G$ (M_P being the reduced Planck mass), equations (12)-(17) reduce to

$$\rho_m = \frac{C_2 2^{2/\alpha} (x^\alpha + 1)^{1/\alpha}}{x^4}, \quad (19)$$

$$p_m = \frac{C_2 c^2 2^{2/\alpha} (x^\alpha + 1)^{(1-\alpha)/\alpha}}{3x^4}, \quad (20)$$

$$\rho_{de} = \frac{C_2 (x^\alpha + 1)^{3/\alpha}}{x^3}, \quad (21)$$

$$p_{de} = -\frac{C_2 c^2 (x^\alpha + 1)^{(3-\alpha)/\alpha}}{x^{(3-\alpha)}}, \quad (22)$$

$$H = \sqrt{\frac{C_2 \hbar c}{3M_P^2 x^4} \left[2^{2/\alpha} (x^\alpha + 1)^{1/\alpha} + x (x^\alpha + 1)^{3/\alpha} \right]}, \quad (23)$$

$$w_{eff} = \frac{-3x^{1+\alpha} + 2^{2/\alpha} (x^\alpha + 1)^{-2/\alpha}}{3(x^\alpha + 1) [x + 2^{2/\alpha} (x^\alpha + 1)^{-2/\alpha}]}. \quad (24)$$

The deceleration, jerk and snap parameters of the model are respectively given by

$$q = -\frac{\ddot{a}}{aH^2} = -1 - x \frac{H'}{H} = \frac{2^{2/\alpha} (x^\alpha + 2) - x(2x^\alpha - 1)(x^\alpha + 1)^{2/\alpha}}{2(x^\alpha + 1) [2^{2/\alpha} + x(x^\alpha + 1)^{2/\alpha}]}, \quad (25)$$

$$j = \frac{\ddot{\ddot{a}}}{aH^3} = 1 + 4x \frac{H'}{H} + x^2 \left[\left(\frac{H'}{H} \right)^2 + \frac{H''}{H} \right], \quad (26)$$

$$s = \frac{\ddot{a}}{aH^4} = 1 + 11x \frac{H'}{H} + x^2 \left[11 \left(\frac{H'}{H} \right)^2 + 7 \frac{H''}{H} \right] + x^3 \left[\left(\frac{H'}{H} \right)^3 + 4 \frac{H' H''}{H^2} + \frac{H'''}{H} \right], \quad (27)$$

where a prime stands for the derivative with respect to x , and H is given by (23).

The matter density parameter (Ω_m) and DE density parameter (Ω_{de}) read as

$$\Omega_m = \frac{\hbar c \rho_m}{3M_P^2 H^2} = \frac{2^{2/\alpha}}{2^{2/\alpha} + x(x^\alpha + 1)^{2/\alpha}}, \quad (28)$$

$$\Omega_{de} = \frac{\hbar c \rho_{de}}{3M_P^2 H^2} = \frac{x(x^\alpha + 1)^{2/\alpha}}{2^{2/\alpha} + x(x^\alpha + 1)^{2/\alpha}}. \quad (29)$$

In view of (6), it is observed that $\Omega_m + \Omega_{de} = 1$.

In order to examine where the unified model stands with respect to the so called standard Λ CDM model, we produce the kinematics of the standard Λ CDM model in Appendix I.

In what follows, we find observational constraints on the parameters of the derived model and Λ CDM model using the following observational results from WMAP7+BAO+ H_0 given in Ref.[14]:

$$H_0 = 70.4_{-1.4}^{+1.3} \text{ km s}^{-1} \text{ Mpc}^{-1}, \quad \Omega_{de0} = 0.728_{-0.016}^{+0.015}.$$

At 1σ level, the parameter Λ in the Λ CDM model is constrained as $\Lambda = (1.25_{-0.07}^{+0.08}) \times 10^{-52} \text{ m}^{-2}$ while the constraints on deceleration parameter of the Λ CDM model are $q_{\Lambda0} = -0.59_{-0.02}^{+0.02}$. WMAP observations suggest that the flat Λ CDM model is most suitable to describe the Universe at the present epoch. So it would be useful to synchronize the derived model with the values of Hubble parameter, deceleration parameter and DE density parameter as given above. With the best fit values of dark energy density parameter and deceleration parameter, we immediately find $\alpha = 18.15$ and $z_* = 0.42$. It may be noted that the values of α and z_* are highly sensitive with respect to the values of Ω_{de} and q in their permissible domains from WMAP. For instance, choosing $\Omega_{de0} = 0.743$ and $q_{\Lambda0} = -0.57$, we obtain $\alpha = 8.06$ and $z_* = 0.50$. However, in what follows, we shall utilize only the best fit values of the parameters derived above.

Table 2 displays the extreme values of various parameters of the derived model along with their best fit values in contrast with those of the Λ CDM model. For the sake of completeness and to examine how the derived model differs from the Λ CDM model, we show the variation of various parameters for the derived model and the Λ CDM model in Fig. 1 to Fig. 8. After a careful and straightforward analysis of the values of various parameters displayed in Table 2 and the graphics (Fig. 1 to Fig. 8), we conclude that the model in hand is not only consistent with the Λ CDM model at the present epoch but also is indistinguishable from the Λ CDM model in revealing the future dynamics of the Universe. Next, it differs significantly from the Λ CDM model at the earlier epochs of evolution dictating its advantages over the usual Λ CDM one. For instance, the Λ CDM model accounts only for pressureless matter even in the early stages of evolution. On the other hand, the derived model successfully describes the evolution of the Universe from the early radiation-dominated matter phase to the current pressureless matter phase (see Fig. 1 and values of w_m in Table 2). One may also observe the significant difference in the values of the parameters q , j and s for the two models (see Table 2 and Fig. 6 to Fig. 8).

Table 2: Extreme and the best fit values of various parameters pertaining to the derived model and the Λ CDM model

Model \rightarrow	Derived Model			Λ CDM		
Parameter	$z \rightarrow \infty$	$z = 0$	$z \rightarrow -1$	$z \rightarrow \infty$	$z = 0$	$z \rightarrow -1$
w_m	1/3	0.0005	0	0	0	0
w_{de}	0	-0.998	-1	-1	-1	-1
w_{eff}	1/3	-0.726	-1	0	-0.728	-1
p_m (10^{-10} Pa)	∞	0.001	0	0	0	0
p_{de} (10^{-10} Pa)	0	-6.073	-6.081	-6.083	-6.083	-6.083
p_{eff} (10^{-10} Pa)	∞	-6.072	-6.081	-6.083	-6.083	-6.083
ρ_m (10^{-27} kg m $^{-3}$)	∞	2.525	0	∞	2.525	0
ρ_{de} (10^{-27} kg m $^{-3}$)	∞	6.759	6.757	6.759	6.759	6.759
ρ_{eff} (10^{-27} kg m $^{-3}$)	∞	9.284	6.757	∞	9.284	6.759
Ω_m	1	0.272	0	1	0.272	0
Ω_{de}	0	0.728	1	0	0.728	1
Ω_{eff}	1	1	1	1	1	1
H (km s $^{-1}$ Mpc $^{-1}$)	∞	70.4	60.1	∞	70.4	60.1
q	1	-0.59	-1	0.5	-0.59	-1
j	3	1.03	1	1	1	1
s	-15	-0.79	1	-3.5	-0.22	1

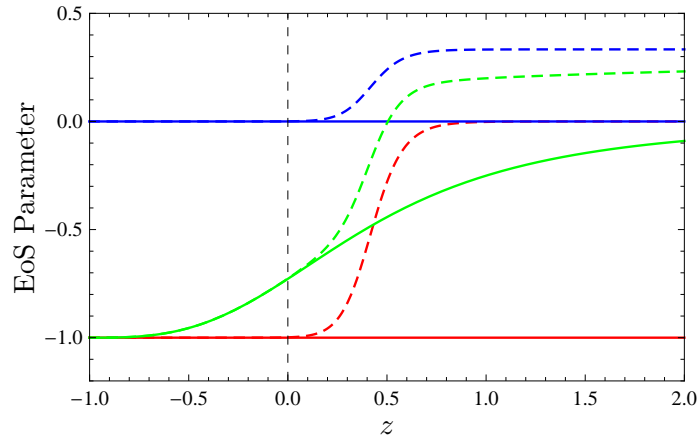


Figure 1: EoS parameters w_m (dashed blue curve), $w_{\Lambda m}$ (solid blue curve), w_{de} (dashed red curve), $w_{\Lambda de}$ (solid red curve), w_{eff} (dashed green curve), $w_{\Lambda eff}$ (solid green curve) vs z . The vertical dashed line is for $z = 0$.

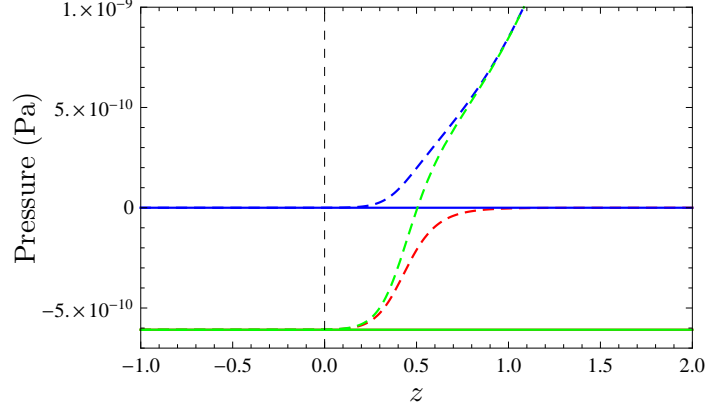


Figure 2: Pressures p_m (dashed blue curve), $p_{\Lambda m}$ (solid blue curve), p_{de} (dashed red curve), $p_{\Lambda de}$ (solid red curve), p_{eff} (dashed green curve), $p_{\Lambda eff}$ (solid green curve) vs z . The vertical dashed line is for $z = 0$. The curves related to $p_{\Lambda de}$ and $p_{\Lambda eff}$ are overlapping as $p_{\Lambda de} = p_{\Lambda eff}$ for all z .

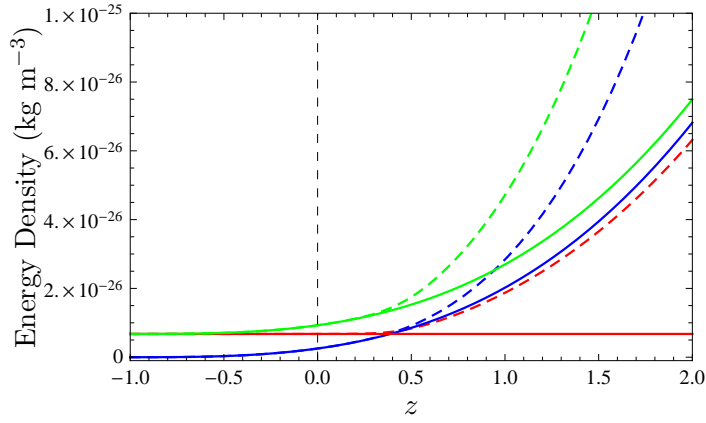


Figure 3: Energy densities ρ_m (dashed blue curve), $\rho_{\Lambda m}$ (solid blue curve), ρ_{de} (dashed red curve), $\rho_{\Lambda de}$ (solid red curve), ρ_{eff} (dashed green curve), $\rho_{\Lambda eff}$ (solid green curve) vs z . The vertical dashed line stands for $z = 0$.

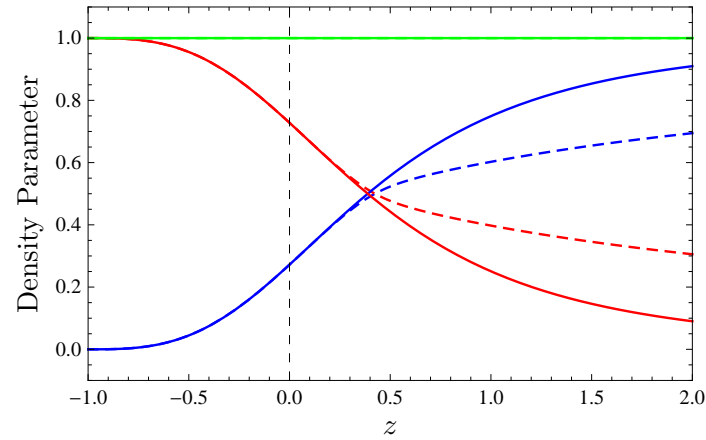


Figure 4: Density parameters Ω_m (dashed blue curve), $\Omega_{\Lambda m}$ (solid blue curve), Ω_{de} (dashed red curve), $\Omega_{\Lambda de}$ (solid red curve), Ω_{eff} (dashed green curve), $\Omega_{\Lambda eff}$ (solid green curve) vs z . The vertical dashed line is for $z = 0$. Since $\Omega_{eff} = \Omega_{\Lambda eff} = 1$ for all z , the green curves related to these parameters are overlapping.

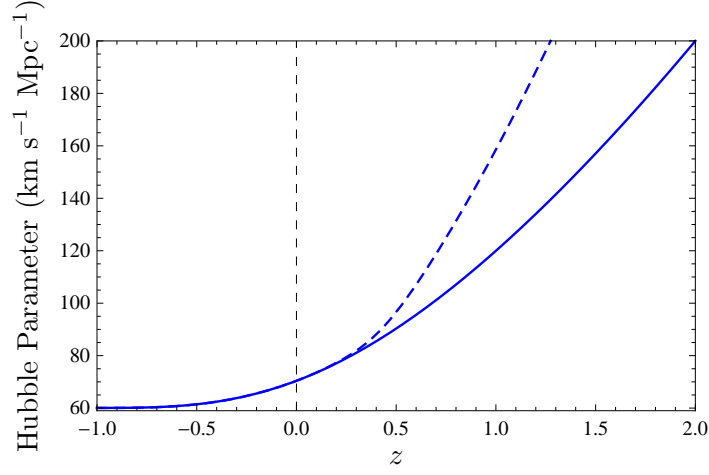


Figure 5: Hubble parameters H (dashed blue curve), H_Λ (solid blue curve) vs z . The vertical dashed line is for $z = 0$.

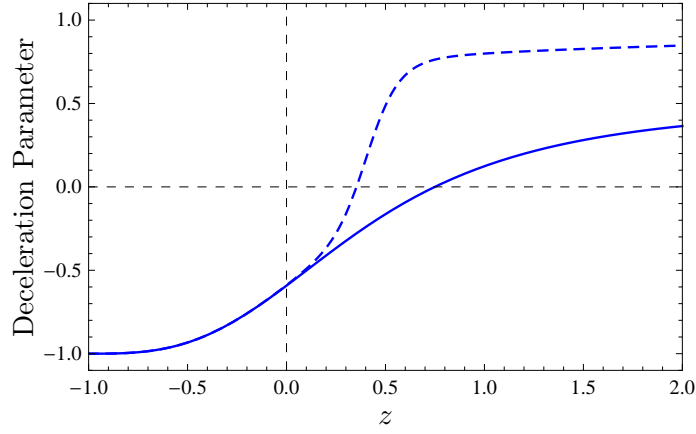


Figure 6: Deceleration parameters q (dashed blue curve), q_Λ (solid blue curve) vs z . The horizontal and vertical dashed lines respectively stand for $q = 0$ and $z = 0$. The transition redshift for the derived model is $z_T = 0.35$ while for the Λ CDM model the transition takes place at $z_T = 0.75$.

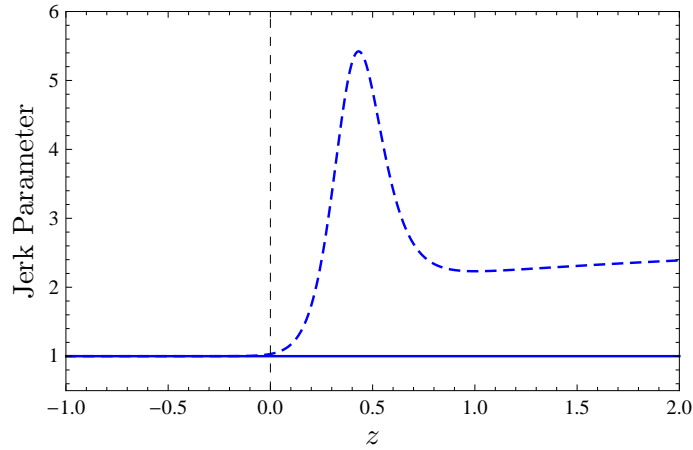


Figure 7: Jerk parameters j (dashed blue curve), j_Λ (solid blue curve) vs z . The vertical dashed line stands for $z = 0$. The jerk parameter j of the derived model attains its maximum value 5.42 at $z = 0.43$.

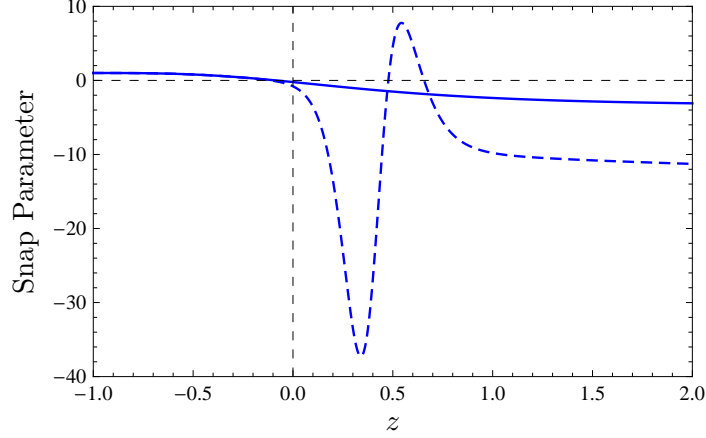


Figure 8: Snap parameters s (dashed blue curve), s_Λ (solid blue curve) vs z . The horizontal and vertical dashed lines respectively stand for $s = 0$ and $z = 0$. The snap parameter s of the derived model attains its maximum value 7.67 at $z = 0.54$ and minimum value -37.26 at $z = 0.33$.

5 Observational constraints on general class of the model

The Friedmann equation of the derived model, in the general case, can be rewritten as

$$\begin{aligned} H^2(a) &= \frac{8\pi G}{3} \left[\rho_{m0} \frac{\{(a^\alpha + a_*^\alpha)/(1 + a_*^\alpha)\}^{1/\alpha}}{a^4} + \rho_{de0} \frac{\{(a^\alpha + a_*^\alpha)/(1 + a_*^\alpha)\}^{-3\bar{w}/\alpha}}{a^3} \right] \\ &= H_0^2 \left[\Omega_{m0} \frac{\{(a^\alpha + a_*^\alpha)/(1 + a_*^\alpha)\}^{1/\alpha}}{a^4} + \Omega_{de0} \frac{\{(a^\alpha + a_*^\alpha)/(1 + a_*^\alpha)\}^{-3\bar{w}/\alpha}}{a^3} \right], \end{aligned} \quad (30)$$

where

$$\Omega_{i0} = \frac{8\pi G \rho_{i0}}{3H_0^2}, \quad \Omega_{m0} + \Omega_{de0} = 1. \quad (31)$$

and a is the scale factor.

To test the viability and to obtain the parameter space of this model, we use SN Ia and BAO data sets and the Markov Chain Monte Carlo (MCMC) method. Our code is based on the publicly available package **cosmoMC** [24]. At first, we modified the code to add three new parameters α , \bar{w} and a_* . The following 4-dimensional parameter space is adopted

$$P \equiv \{w_c, \alpha, \bar{w}, a_*\} \quad (32)$$

where $w_c = \Omega_c h^2$ is the physical cold dark matter density. We take the following priors to model parameters: $w_c \in [0.01, 0.99]$, $\alpha \in (0, 100]$, $a_* \in [0, 0.1]$ and $\bar{w} \in [-3, 0]$. In addition, the hard coded prior on the comoving age $10\text{Gyr} < t_0 < 20\text{Gyr}$ is imposed. Also, we fixed the physical baryon density $\omega_b = 0.022$ [25] from big bang nucleosynthesis and the new Hubble constant $H_0 = 74.2 \pm 3.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [26].

To get the distribution of parameters, we calculate the total likelihood $\mathcal{L} \propto e^{-\chi^2/2}$, where χ^2 is given as

$$\chi^2 = \chi_{BAO}^2 + \chi_{SN}^2. \quad (33)$$

The 557 Union2 data [27] with systematic errors and BAO [28, 29] are used to constrain the background evolution. For the detailed description, see Refs. [30].

After running 8 independent chains and checking the convergence to stop sampling when the worst e-values [the variance(mean)/mean(variance) of 1/2 chains] $R - 1$ is of the order 0.01, the global fitting results are summarized in Table 3 and Fig. 9. For the sake of comparison and to see the viability

of the derived results, we also show the values of various parameters in Table 3 based on WMAP and WMAP+BAO+H0 observations (see Jarosik et al. [14]).

One may see that the derived model is in close agreement with the results predicted by WMAP and WMAP+BAO+H0 observations. It may be noted that the values of α can be taken in a large range, and current cosmic observations from SN Ia and BAO cannot give a tight constraint to the parameter α . For, smaller values of a_* and values of $\alpha \geq 1$, in view of (30), lead to the DE model with EoS $w = \bar{w}$ via

$$H^2(a) \approx H_0^2 \left[\Omega_{m0} a^{-3} + \Omega_{de0} a^{-3(1+\bar{w})} \right]. \quad (34)$$

Further, in view of (10) it deserves mention that smaller values of a_* and values of $\alpha \geq 1$ yield values of w_m closer to 0. This in turn implies that the observations from SN Ia and BAO force the model to describe the evolution of the Universe from relatively later phase of radiation to the current phase dominated by some sort of DE with EoS $\bar{w} \approx -1$. Thus, the model puts forward cosmological constant as a candidate of DE. This is consistent with the WMAP observations.

Table 3: Best fit values of the derived model parameters along with error bars at 68% and 95% levels, and values of some relevant parameters based on WMAP and WMAP+BAO+H0 observations

Parameter	SN Ia+BAO (This paper)	WMAP (Jarosik et al. [14])	WMAP+BAO+H0 (Jarosik et al. [14])
α	$42.832^{+15.164+37.349}_{-42.832-42.832}$	—	—
a_*	$0.000138^{+0.000038+0.000109}_{-0.000138-0.000138}$	—	—
Ω_m	$0.287^{+0.0200+0.0427}_{-0.0197-0.0368}$	—	—
$\Omega_c h^2$	$0.133^{+0.0115+0.0254}_{-0.0112-0.0188}$	$0.1109^{+0.0056}_{-0.0056}$	$0.1123^{+0.0035}_{-0.0035}$
Ω_Λ	$0.713^{+0.0197+0.0368}_{-0.0200-0.0427}$	$0.734^{+0.029}_{-0.029}$	$0.728^{+0.015}_{-0.016}$
\bar{w}	$-1.0758^{+0.0943+0.186}_{-0.0944-0.186}$	$-1.12^{+0.42}_{-0.43}$	$-0.980^{+0.053}_{-0.053}$
Age (Gyr)	$13.119^{+0.499+0.640}_{-0.480-0.983}$	$13.75^{+0.13}_{-0.13}$	$13.75^{+0.11}_{-0.11}$
H_0 (km s ⁻¹ Mpc ⁻¹)	$73.617^{+2.870+5.990}_{-2.775-5.0607}$	$71.0^{+2.5}_{-2.5}$	$70.4^{+1.3}_{-1.4}$

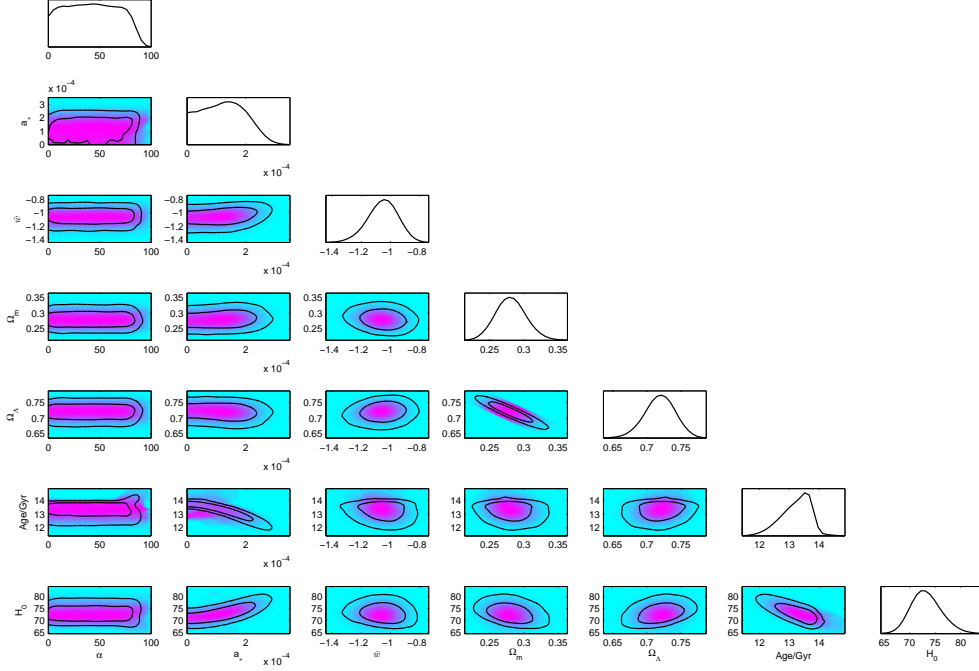


Figure 9: The 1D marginalized distribution on individual parameters and 2D contours with 68% and 95% confidence limits obtained by using SN Ia+BAO data points. The shaded regions show the mean likelihood of the samples.

6 Concluding remarks

In this work, we have investigated a cosmological model within the framework of a spatially homogeneous and flat FRW space-time filled with non-interacting matter and DE components. Following Ref. [18], the model is derived by assuming time-varying EoS parameters of the two sources, which in turn provides an elegant evolution of the Universe from the early radiation-dominated phase to the present DE-dominated phase. We have explored a special case of the model with WMAP+BAO+H0 observations by synchronizing the model with the Λ CDM model at the present epoch. The observational analysis suggests that the model is not only consistent with the Λ CDM predictions at the present epoch but also is indistinguishable from the Λ CDM model in revealing the future dynamics of the Universe. Thus the derived model, in the special case, has already revealed what Λ CDM model has to offer. In addition, it accounts for radiation-dominated matter phase at early epochs. Thus, the derived model has an advantage over the usual Λ CDM one. We have also tested the viability of the general class of the model by constraining the model with SN Ia and BAO data sets. In the general case also the derived model yields parameters consistent with the WMAP and WMAP+BAO+H0 observations. The model advocates cosmological constant as a candidate of DE, which is consistent with the WMAP observations. Finally, we conclude that the derived model offers a unified description of the evolution of Universe from the early radiation-dominated phase to the present DE-dominated phase in accord with the current astronomical observations. The model is applicable to the real Universe, and is supposed to yield more accurate results with the advancement of cosmic data. It would be interesting to find observational constraints on the cosmological model based on generalized EoS parameters for matter and DE reported in Ref. [18].

Acknowledgments

S.K. acknowledges the warm hospitality and research facilities provided by the Inter-University Centre for Astronomy and Astrophysics (IUCAA), India where a part of this work was carried out. The authors are thankful to J. Ponce de Leon for his valuable comments on the initial draft of the paper.

References

- [1] A.G. Riess et al., *Astron. J.* **116**, 1009 (1998)
- [2] S. Perlmutter et al., *Astrophys. J.* **517**, 565 (1999)
- [3] C. Fedeli, L. Moscardini and M. Bartelmann *Astron. Astrophys.* **500**, 667 (2009)
- [4] R.R. Caldwell and M. Doran, *Phys. Rev. D* **69**, 103517 (2004)
- [5] Z-Yi. Huang, B. Wang, E. Abdalla and Ru-K. Sul, *JCAP* **05**, 013 (2006)
- [6] S.F. Daniel, R.R. Caldwell, A. Cooray and A. Melchiorri, *Phys. Rev. D* **77**, 103513 (2008)
- [7] R.R. Caldwell, W. Komp, L. Parker and D.A.T. Vanzella, *Phys. Rev. D* **73**, 023513 (2006)
- [8] P. J. E. Peebles and B. Ratra, *Rev. Mod. Phys.* **75**, 559 (2003)
- [9] G. Hinshaw et al., *Astrophys. J. Suppl.* **180**, 225 (2009)
- [10] E. J. Copeland, M. Sami, S. Tsujikava, *Int. J. Mod. Phys. D* **15**, 1753 (2006)
- [11] P.J. Steinhardt, L.M. Wang and I. Zlatev, *Phys. Rev. D* **59**, 123504 (1999)
- [12] R.R. Caldwell, *Phys. Lett. B* **545**, 23 (2002)
- [13] H. Lampeitl et al., *MNRAS* **401**, 2331 (2009)
- [14] N. Jarosik et al., *Astrophys. J. Suppl.* **192**, 14 (2011)
- [15] M. Israelit and N. Rosen, *Astrophys. J.* **342**, 627 (1989)
- [16] M. Israelit and N. Rosen, *Astrophys. Space Sci.* **204**, 317 (1993)
- [17] J. C. Carvalho, *Int. J. Theor. Phys.* **35**, 2019 (1996)
- [18] J. Ponce de Leon, *Class. Quantum Grav.* **29**, 135009 (2012)
- [19] R. Amanullah et al., *Astrophys. J.* **716**, 712 (2010)
- [20] A. Avelino and U. Nucamendi, *JCAP* **04**, 006 (2009)
- [21] A.G. Riess et al., *Astron. J.* **607**, 665 (2004)
- [22] J.V. Cunha, *Phys. Rev. D* **79**, 047301 (2009)
- [23] A.M.V. Toribio and M.L. Bedran , *Braz. J. Phys.* **41**, 59 (2011)
- [24] <http://cosmologist.info/cosmomc/>; A. Lewis and S. Bridle, *Phys. Rev. D* **66**, 103511 (2002).
- [25] S. Burles, K. M. Nollett, and M. S. Turner, *Astrophys. J.* **552**, L1 (2001).

- [26] A. G. Riess et al., *Astrophys. J.* **699**, 539 (2009).
- [27] R. Amanullah et al. (Supernova Cosmology Project Collaboration), *Astrophys. J.* **716**, 712 (2010).
- [28] W. J. Percival et al., *MNRAS* **401**, 2148 (2010).
- [29] C. Blake, et al., arXiv:1108.2635[astro-ph.CO].
- [30] L. Xu, Y. Wang, *JCAP* **06**, 002(2010); L. Xu, Y. Wang, *Phys. Rev. D* **82**, 043503 (2010); L. Xu, *Phys. Rev. D* **85**, 123505 (2012); S. Kumar, *MNRAS* **422**, 2532 (2012).

Appendix I. Elements of Λ CDM cosmology

The standard Λ CDM Universe is governed by the scale factor

$$a_\Lambda = a_1 \sinh^{\frac{2}{3}} \left(\sqrt{\frac{3\Lambda c^2}{4}} t \right), \quad (35)$$

where a_1 is a constant.

Using the relation $a_\Lambda = a_0/(1+z)$, the Hubble parameter, deceleration parameter, jerk and snap parameters in terms of the redshift are obtained as

$$H_\Lambda = \sqrt{\frac{\Lambda c^2 [a_1^3(1+z)^3 + a_0^3]}{3a_0^3}}, \quad (36)$$

$$q_\Lambda = \frac{-2a_0^3 + a_1^3(1+z)^3}{2[a_0^3 + a_1^3(1+z)^3]}, \quad (37)$$

$$j_\Lambda = 1, \quad (38)$$

$$s_\Lambda = \frac{2a_0^3 - 7a_1^3(1+z)^3}{2[a_0^3 + a_1^3(1+z)^3]}. \quad (39)$$

The pressure and energy density of the ordinary matter in Λ CDM cosmology are

$$p_{\Lambda m} = 0, \quad (40)$$

$$\rho_{\Lambda m} = \frac{M_P^2 \Lambda c a_1^3 (1+z)^3}{\hbar a_0^3}, \quad (41)$$

while the pressure and density of the vacuum energy associated with the cosmological constant read as

$$p_{\Lambda de} = -\frac{M_P^2 \Lambda c^3}{\hbar}, \quad (42)$$

$$\rho_{\Lambda de} = \frac{M_P^2 \Lambda c}{\hbar}. \quad (43)$$

The density parameters and effective EoS parameter in the Λ CDM cosmology is given by

$$\Omega_{\Lambda m} = \frac{a_1^3(1+z)^3}{a_0^3 + a_1^3(1+z)^3}, \quad (44)$$

$$\Omega_{\Lambda de} = \frac{a_0^3}{a_0^3 + a_1^3(1+z)^3}, \quad (45)$$

$$w_{\Lambda eff} = -\frac{a_0^3}{a_0^3 + a_1^3(1+z)^3}. \quad (46)$$

Obviously, the EoS parameters of ordinary matter and vacuum energy in the Λ CDM Universe are $w_{\Lambda m} = 0$ and $w_{\Lambda de} = -1$, respectively.